Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

Kiminori Matsuyama
Northwestern University

Philip Ushchev
ECARES, Université Libre de Bruxelles

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Structure of the Talk

- Introduction
- Monopolistic Competition under H.S.A.
- Selection of Heterogenous Firms: A Single Market Setting
    - Existence and Uniqueness
    - CES Benchmark: Revisiting Melitz.
    - Cross-Sectional Implications of the 2^{nd} & 3rd Laws
    - Comparative Statics: General Equilibrium Effects
- Sorting of Heterogenous Firms Across Multiple Markets
- International/Interregional Trade with Differential Market Access.

Appendix: Some Parametric Families of H.S.A.

Introduction

Competitive Pressures on Heterogeneous Firms

Main Questions: How do more ***competitive pressures***, caused by lower ***entry cost***, larger ***market size***, or ***globalization*** affect firms with different productivity?

- Selection of firms
- Distribution of firm size (in revenue, profit and employment), Distribution of markup and pass-through rates, etc.
- Sorting of firms across markets with different market sizes

Existing Monopolistic Competition Models with Heterogenous Firms

- Melitz (2003): under **CES** Demand System (DS)
  - MC firms sell their products at an exogenous & common markup rate, *unresponsive to competitive pressures, inconsistent with some evidence for*
    - A higher production cost leads to less than proportional increase in the price (the pass-through rate < 1)
    - More productive firms have higher markup rates and lower pass-through rates
  - Firm size distribution does not depend on whether it is measured in revenue, profit, or in employment.
  - Market size: no effect on distribution of firm types nor their behaviors; All adjustments at *the extensive margin*.
  - Firms' incentive to move across markets with different market sizes independent of firm productivity
- Melitz-Ottaviano (2008) departs from CES with **Linear Demand System + the outside competitive sector**, which comes with its own restrictions.

This Paper: Melitz under **H.S.A. (Homothetic Single Aggregator)** Demand System to study how departing from CES in the direction consistent with the evidence affects the impact of competitive pressures on heterogeneous firms.

Symmetric H.S.A. (Homothetic Single Aggregator) DS with Gross Substitutes

Think of a competitive final goods industry generating demand for a continuum of **intermediate inputs** $\omega \in \Omega$, with
CRS production function: $X = X(\mathbf{x})$; $\mathbf{x} = \{x_\omega;\ \omega \in \Omega\} \Leftrightarrow$ **Unit cost function,** $P = P(\mathbf{p})$; $\mathbf{p} = \{p_\omega;\ \omega \in \Omega\}$.

Market share of ω depends *solely* on a single variable, its own price normalized by the *common* price aggregator

$$s_\omega \equiv \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right), \qquad \text{where} \qquad \int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$$

- $s: \mathbb{R}_{++} \to \mathbb{R}_+$: **the market share function**, C^3, decreasing in the **normalized price** $z_\omega \equiv p_\omega / A$ for $s(z_\omega) > 0$ with
  - $\lim_{z \to \bar{z}} s(z) = 0$. If $\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty$, $\bar{z}A(\mathbf{p})$ is the **choke price.**
- $A = A(\mathbf{p})$: the **common price aggregator** defined implicitly by **the adding-up constraint** $\int_\Omega s(p_\omega / A) d\omega \equiv 1$.
  $A(\mathbf{p})$ linear homogenous in $\mathbf{p}$ for a fixed Ω. A larger Ω reduces $A(\mathbf{p})$.

Special Cases	**CES**	$s(z) = \gamma z^{1-\sigma}$;	$\sigma > 1$
	Translog Cost Function	$s(z) = \gamma \max\{-\ln(z/\bar{z}), 0\}$;	$\bar{z} < \infty$
	Constant Pass Through (CoPaTh)	$s(z) = \gamma \max\left\{\left[\sigma + (1-\sigma) z^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}, 0\right\}$	$0 < \rho < 1$

As $\rho \nearrow 1$, CoPaTh converges to CES with $\bar{z}(\rho) \equiv (\sigma/(\sigma-1))^{\frac{\rho}{1-\rho}} \to \infty$.

$P(\mathbf{p})$ vs. $A(\mathbf{p})$

Definition: $s_\omega \equiv \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\dfrac{p_\omega}{A(\mathbf{p})}\right) = s(z_\omega)$ where $\displaystyle\int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$

By differentiating the adding-up constraint, $\dfrac{\partial \ln A(\mathbf{p})}{\partial \ln p_\omega} = \dfrac{[\zeta(z_\omega)-1]s(z_\omega)}{\int_\Omega [\zeta(z_{\omega'})-1]s(z_{\omega'})d\omega'} \neq s(z_\omega) = \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega}$

unless $\zeta(z_\omega)$ is constant, where

Price Elasticity Function: $\zeta(z) \equiv 1 - \dfrac{d\ln s(z)}{d\ln z} \equiv 1 - \mathcal{E}_s(z) > 1 \Leftrightarrow s(z) = \gamma \exp\left[\displaystyle\int_{z_0}^{z} \frac{1-\zeta(\xi)}{\xi} d\xi\right];\ \lim_{z\to\bar{z}} \zeta(z) = \infty, \text{if } \bar{z} < \infty.$

By integrating the definition, $\dfrac{A(\mathbf{p})}{P(\mathbf{p})} = c \exp\left[\displaystyle\int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) \Phi\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega\right],$ where $\Phi(z) \equiv \dfrac{1}{s(z)}\displaystyle\int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi$

$c > 0$: The integral constant, proportional to TFP. $P(\mathbf{p})$ clearly satisfies linear homogeneity, monotonicity, and symmetry. Our 2017 paper proved the quasi-concavity of $P(\mathbf{p})$, iff $\zeta(z) \equiv 1 - \frac{d\ln s(z)}{d\ln z} \equiv 1 - \mathcal{E}_s(z) > 0$.

Note: $A(\mathbf{p})/P(\mathbf{p})$ is not constant, unless CES $\Leftrightarrow \zeta(z) = \sigma \Leftrightarrow s(z) = \gamma z^{1-\sigma} \Leftrightarrow \Phi(z) = 1/(\sigma-1)$.

- ✓ $A(\mathbf{p})$, the inverse measure of *competitive pressures*, captures *cross price effects* in the DS, the reference price for MC firms
- ✓ $P(\mathbf{p})$, the inverse measure of TFP, captures the *productivity effects* of price changes, the reference price for consumers.
- ✓ $\Phi(z) > 0$, Productivity gains from a product sold at $z > 0$. $\zeta'(\cdot) \gtreqless 0 \Rightarrow \Phi'(\cdot) \lesseqgtr 0$; $\Phi'(\cdot) = 0 \Leftrightarrow \zeta'(\cdot) = 0$. The measure of "love for variety." Matsuyama & Ushchev (2023)..

Why H.S.A.

- **Homothetic** (unlike the linear DS and most other commonly used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block in multi-sector models with any upper-tier (incl. nonhomothetic) DS

- **Nonparametric** and **flexible** (unlike **CES** and **translog**, which are special cases)
  - can be used to perform robustness-check for CES
  - allow for (but no need to impose)
    - ✓ the choke price,
    - ✓ **Marshall's 2nd law** (Price elasticity is increasing in price) → more productive firms have higher markup rates
    - ✓ *what we call* **the 3rd law** (the rate of increase in the price elasticity is decreasing in price) → more productive firms have lower pass-through rates.

- **Tractable** due to **Single Aggregator** (unlike **Kimball**, which needs two aggregators), a *sufficient statistic* for competitive pressures, which acts like a *magnifier of firm heterogeneity*
  - guarantee the existence & uniqueness of free-entry equilibrium with firm heterogeneity
  - simple to conduct most comparative statics without *parametric* restrictions on demand or productivity distribution.
  - no need to assume zero overhead cost (unlike MO and ACDR)

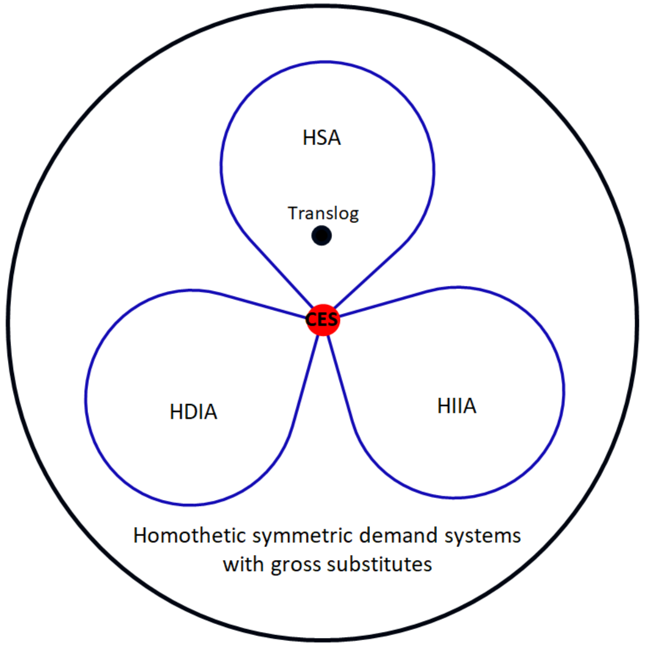
- Defined by **the market share function,** for which data is readily available and easily identifiable.

Three Classes of Homothetic Demand Systems: Matsuyama-Ushchev (2017)

Here we consider a **continuum** of varieties ($\omega \in \Omega$), **gross substitutes**, and **symmetry**

CES	$s_\omega \equiv \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = f\left(\frac{p_\omega}{P(\mathbf{p})}\right) \Leftrightarrow s_\omega \propto \left(\frac{p_\omega}{P(\mathbf{p})}\right)^{1-\sigma}$	
H.S.A. (Homotheticity with a Single Aggregator)	$s_\omega = s\left(\frac{p_\omega}{A(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{A(\mathbf{p})} \neq c$, unless CES
HDIA (Homotheticity with Direct Implicit Additivity) Kimball is a special case:	$s_\omega = \frac{p_\omega}{P(\mathbf{p})}(\phi')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{B(\mathbf{p})} \neq c$, unless CES
HIIA (Homotheticity with Indirect Implicit Additivity)	$s_\omega = \frac{p_\omega}{C(\mathbf{p})}\theta'\left(\frac{p_\omega}{P(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{C(\mathbf{p})} \neq c$, unless CES

$\phi(\cdot)$ & $\theta(\cdot)$ are both increasing & concave → $(\phi')^{-1}(\cdot)$ & $\theta'(\cdot)$ positive-valued & decreasing.
$A(\cdot), B(\cdot), C(\cdot)$ all determined by the adding-up constraint.



The 3 classes are pairwise disjoint with the sole exception of CES.

We use HSA, because, under HDIA(Kimball) and HIIA,

- Two aggregators needed for the market shares. [One aggregator enough for the price elasticity under all 3 classes.]
- The existence and uniqueness of free-entry equilibrium not guaranteed without some strong restrictions on both productivity distribution and the price elasticity function.

Note: Beyond these three, "almost anything goes." E.g., Marshall's 2nd Law doesn't ensure even procompetitive entry.

A Summary of Main Results

- **Existence & Uniqueness of Equilibrium:** straightforward under H.S.A., not under HDIA/HIIA.
- **Under CES (i.e., Melitz),** we not only reproduce well-known results but also have some new results.
  - Impacts of entry/overhead costs on the masses of entrants/active firms hinges on the sign of the derivative of the elasticity of the pdf of marginal cost. Pareto is the knife-edge!
- **Cross-Sectional Implications:** profits and revenues are always higher among more productive.
  - 2nd Law = incomplete pass-through ⇔ the procompetitive effect ⇔ strategic complementarity in pricing.
  - 2nd (3rd) Law → more productive firms have higher markup (lower pass-through) rates.
  - 2nd & 3rd Laws → *hump-shaped* employment; the more productive hire less labor under high overhead cost..
- **Comparative Statics**
  - *Entry cost* ↓: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) decline faster among less productive → a tougher selection.
  - *Overhead cost* ↓: similar effects when the employment is decreasing in firm productivity.
  - *Market size* ↑: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) ↑ among more productive; ↓ among less productive.
  - *Composition effect*, these changes may *increase* the average markup rate & the aggregate profit share in spite of 2nd Law and *reduce* the average pass-through in spite of 3rd Law; Pareto is the knife-edge *for entry cost* ↑.
- **Sorting of Heterogeneous Firms** across markets that differ in size: Larger markets→ more competitive pressures.
  - 2nd Law → more (less) productive go into larger (smaller) markets.
  - *Composition effect*, average markup (pass-through) rates can be *higher* (*lower*) in larger markets in spite of 2nd (3rd) Law.
- **International Trade with Differential Market Access**
  - 2nd Law → Exporters sell their products at lower markup rates abroad than at home.
  - Globalization (Iceberg cost ↓) → share of exporting (domestic) firms up (down);
  - Exporters reduce the markup rate at home, increases their markup rate abroad.

(Highly Selective) Literature Review

Non-CES Demand Systems: Matsuyama (2023) for a survey; **H.S.A. Demand System:** Matsuyama-Ushchev (2017)

MC with Heterogeneous Firms: Melitz (2003) and many others: Melitz-Redding (2015) for a survey

MC under non-CES demand systems: Thisse-Ushchev (2018) and Matsuyama (2025) for a survey

- *Nonhomothetic non-CES:*
  - $U = \int_{\Omega} u(x_\omega) d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)
  - *Linear-demand system with the outside sector:* Ottaviano-Tabuchi-Thisse (2002), Melitz-Ottaviano (2008)
- *Homothetic non-CES:* Feenstra (2003), Kimball (1995), Matsuyama-Ushchev (2020a,b, 2023)
- *H.S.A.* Matsuyama-Ushchev (2022), Kasahara-Sugita (2020), Grossman-Helpman-Lhuiller (2021), Fujiwara-Matsuyama (2022), Baqaee-Fahri-Sangani (2023)

Empirical Evidence: *The 2nd Law:* DeLoecker-Goldberg (14), Burstein-Gopinath (14); *The 3rd Law:* Berman et.al.(12), Amiti et.al.(19), *Market Size Effects:* Campbell-Hopenhayn(05); *Rise of markup:* Autor et.al.(20), DeLoecker et.al.(20)

Selection of Heterogeneous Firms through Competitive Pressures
Melitz-Ottaviano (2008), Baqaee-Fahri-Sangani (2023), Edmond-Midrigan-Xu (2023)

Sorting of Heterogeneous Firms Across Markets:

- *Reduced Form/Partial Equilibrium;* Mrázová-Neary (2019), Nocke (2006)
- *General Equilibrium:* Baldwin-Okubo (2006), Behrens-Duranton-RobertNicoud (2014), Davis-Dingel (2019), Gaubert (2018), Kokovin et.al. (2022)

Log-Super(Sub)modularity: Costinot (2009), Costinot-Vogel (2015)

Monopolistic Competition under H.S.A.

Pricing: Markup & Pass-Through Rates.

Taking the value of $A = A(\mathbf{p})$ given, firm ω chooses p_ω.

Lerner Pricing Formula

$$p_\omega\left[1-\frac{1}{\zeta(p_\omega/A)}\right]=\psi_\omega \Longrightarrow z_\omega\left[1-\frac{1}{\zeta(z_\omega)}\right]=\frac{\psi_\omega}{A},$$

ψ_ω: *firm-specific* (quality-adjusted) marginal cost (in labor, the numeraire)

Under **(A1),** LHS is strictly increasing in z_ω → firms with the same ψ set the same price → $p_\omega = p_\psi$.

Normalized Price:

$$\frac{p_\psi}{A}\equiv z_\psi = Z\left(\frac{\psi}{A}\right)\in(\psi/A,\bar{z}), Z'(\cdot)>0;$$

Price Elasticity:

$$\zeta(z_\psi)=\zeta\left(Z\left(\frac{\psi}{A}\right)\right)\equiv\sigma\left(\frac{\psi}{A}\right)>1;$$

Markup Rate:

$$\mu_\psi\equiv\frac{p_\psi}{\psi}=\frac{\sigma(\psi/A)}{\sigma(\psi/A)-1}\equiv\mu\left(\frac{\psi}{A}\right)>1$$

$$\Longrightarrow \frac{1}{\sigma(\psi/A)}+\frac{1}{\mu(\psi/A)}=1 \Leftrightarrow \left[\sigma\left(\frac{\psi}{A}\right)-1\right]\left[\mu\left(\frac{\psi}{A}\right)-1\right]=1$$

Pass-Through Rate:

$$\rho_\psi\equiv\frac{\partial\ln p_\psi}{\partial\ln\psi}=\mathcal{E}_Z\left(\frac{\psi}{A}\right)\equiv\rho\left(\frac{\psi}{A}\right)=1+\mathcal{E}_\mu\left(\frac{\psi}{A}\right)=1-\frac{\mathcal{E}_\sigma(\psi/A)}{\sigma(\psi/A)-1}>0$$

are all functions of the *normalized cost,* ψ/A, *only*; continuously differentiable.

- Market size $E = \mathbf{px}$ affects the pricing behaviors of firms only through its effects on A.
- More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

Under CES, $\sigma(\cdot)=\sigma$; $\mu(\cdot)=\sigma/(\sigma-1)=\mu$; $\rho(\cdot)=1$.

Revenue, Profit, & Employment

Revenue	(Gross) Profit	(Variable) Employment
$R_\psi = s(z_\psi)E = s\left(\tilde{Z}\left(\frac{\psi}{A}\right)\right)E \equiv r\left(\frac{\psi}{A}\right)E$	$\Pi_\psi = \frac{r(\psi/A)}{\sigma(\psi/A)}E \equiv \pi\left(\frac{\psi}{A}\right)E$	$\psi x_\psi = \frac{r(\psi/A)}{\mu(\psi/A)}E \equiv \ell\left(\frac{\psi}{A}\right)E$
$\frac{\partial \ln R_\psi}{\partial \ln(\psi/A)} \equiv \mathcal{E}_r\left(\frac{\psi}{A}\right) = \left[1-\sigma\left(\frac{\psi}{A}\right)\right]\rho\left(\frac{\psi}{A}\right) < \mathbf{0}$ Always strictly negative.	$\frac{\partial \ln \Pi_\psi}{\partial \ln(\psi/A)} \equiv \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = 1-\sigma\left(\frac{\psi}{A}\right) < \mathbf{0}$ Always strictly negative.	$\frac{\partial \ln(\psi x_\psi)}{\partial \ln(\psi/A)} \equiv \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1-\sigma\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) ??\mathbf{0}$ Nonmonotone in general.
$\frac{\partial^2 \ln R_\psi}{\partial\psi\partial(1/A)} = \left[1-\sigma\left(\frac{\psi}{A}\right)\right]\rho'\left(\frac{\psi}{A}\right) - \sigma'\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right)$ Negative under the 2nd & weak 3rd laws	$\frac{\partial^2 \ln \Pi_\psi}{\partial\psi\partial(1/A)} = -\sigma'\left(\frac{\psi}{A}\right)$ Negative under the 2nd law	$\frac{\partial^2 \ln(\psi x_\psi)}{\partial\psi\partial(1/A)} = -\sigma'\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) - \sigma\left(\frac{\psi}{A}\right)\rho'\left(\frac{\psi}{A}\right)$ Negative under the 2nd & the weak 3rd laws

- Revenue $r(\psi/A)E$, profit $\pi(\psi/A)E$, employment $\ell(\psi/A)E$ all functions of ψ/A, multiplied by **market size** E, continuously differentiable under mild regularity conditions.
- Their elasticities $\mathcal{E}_r(\cdot), \mathcal{E}_\pi(\cdot)$ and $\mathcal{E}_\ell(\cdot)$ depend solely on $\sigma(\cdot)$ and $\rho(\cdot)$.

  More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

  Market size affects the distribution of the profit, revenue and employment across firms only via its effects on A.

Under CES, $r(\cdot)/\pi(\cdot) = \sigma$; $r(\cdot)/\ell(\cdot) = \mu = \sigma/(\sigma-1) \Longrightarrow \mathcal{E}_r(\cdot) = \mathcal{E}_\pi(\cdot) = \mathcal{E}_\ell(\cdot) = 1-\sigma < 0$.

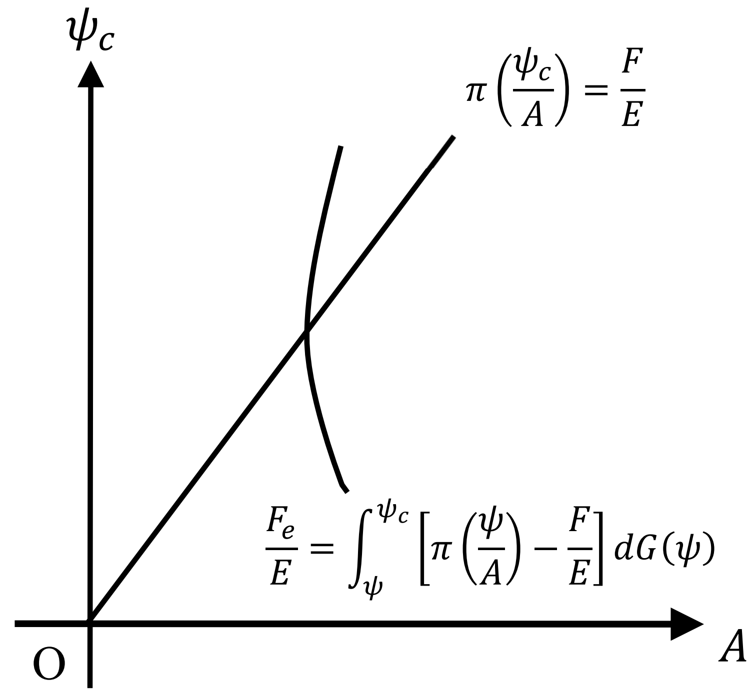
- Both revenue and profit are always strictly decreasing in ψ/A.
- Employment may be nonmonotonic in ψ/A.

Selection of Heterogenous Firms: A Single-Market Setting

General Equilibrium: Existence & Uniqueness

- Ex-ante identical firms pay the entry cost $F_e > 0$ to draw $\psi \sim G(\psi)$, cdf whose support, $\left(\underline{\psi}, \bar{\psi}\right) \subset (0, \infty)$,
- After learning ψ, decide whether to pay the overhead $F > 0$ to stay & produce.

Assume $F + F_e < \pi(0)E$. Otherwise, no firm would enter.



Cutoff Rule: stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\max_{\psi_c} \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right) E - F\right] dG(\psi) \Longrightarrow \pi\left(\frac{\psi_c}{A}\right) E = F$$

positive-sloped, as $A \downarrow$ (more competitive pressures) $\Rightarrow \psi_c \downarrow$ (tougher selection).
rotate clockwise, as $F/E \uparrow$ (higher overhead/market size) $\Longrightarrow \psi_c/A \downarrow$.

Free Entry Condition:

$$F_e = \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right) E - F\right] dG(\psi)$$

shift to the left as $F_e \downarrow$ (lower entry cost) $\Longrightarrow A \downarrow$ (more competitive pressures).

Notes:

- $A = A(\mathbf{p})$ and ψ_c: uniquely determined as functions of F_e/E & F/E, with the interior solution, $0 < G(\psi_c) < 1$, ensured for a sufficiently small $F_e > 0$ with no further restrictions on $G(\cdot)$ and $s(\cdot)$.
- **A sector-wide productivity shock, $G(\psi) \to G(\psi/\lambda)$**: causes $\psi_c \to \lambda\psi_c$, $A \to \lambda A$, leaving ψ_c/A, hence, the markup and the pass-through rates, the profit, the revenue, and the employment distributions across firms all unchanged.

Equilibrium Mass of Firms: With A & ψ_c already determined, from **the Adding-up Constraint,**

Mass of Active Firms
= the measure of Ω.

$$MG(\psi_c) = \left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right)\frac{dG(\psi)}{G(\psi_c)}\right]^{-1} = \left[\int_{\underline{\xi}}^{1} r\left(\frac{\psi_c}{A}\xi\right)d\tilde{G}(\xi;\psi_c)\right]^{-1} > 0$$

where

$$\tilde{G}(\xi;\psi_c) \equiv \frac{G(\psi_c\xi)}{G(\psi_c)}$$

is the cdf of $\xi \equiv \psi/\psi_c$, conditional on $\underline{\xi} \equiv \underline{\psi}/\psi_c < \xi \leq 1$.

Lemma 1: $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0$; $\mathcal{E}_g'(\psi) \geq 0 \Longrightarrow \mathcal{E}_G'(\psi) \geq 0$, with some boundary conditions.
Lemma 2: A lower ψ_c shifts $\tilde{G}(\xi;\psi_c)$ to the right (left) in MLR if $\mathcal{E}_g'(\psi) < (>)0$ and in FSD if $\mathcal{E}_G'(\psi) < (>)0$.

- Some evidence for $\mathcal{E}_g'(\psi) > 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the left.
- Pareto-productivity, $G(\psi) = (\psi/\bar{\psi})^{\kappa} \Longrightarrow \mathcal{E}_g'(\psi) = \mathcal{E}_G'(\psi) = 0 \Longrightarrow \tilde{G}(\xi;\psi_c)$ is independent of ψ_c.
- Fréchet, Weibull, Lognormal; $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the right.

Equilibrium can be solved recursively under H.S.A.!!
Under HDIA/HIIA, all 3 conditions need to be solved simultaneously → possibility of multiplicity/non-existence. (This unique existence proof applies also to the Melitz model.)

Aggregate Labor Cost and Profit Shares and TFP

Notations:

The $w(\cdot)$-weighted average of $f(\cdot)$ among the active firms, $\psi \in (\underline{\psi}, \psi_c)$	$\mathbb{E}_w(f) \equiv \dfrac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) w\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} w\left(\frac{\psi}{A}\right) dG(\psi)}.$
The unweighted average of $f(\cdot)$ among the active firms, $\psi \in (\underline{\psi}, \psi_c)$	$\mathbb{E}_1(f) \equiv \dfrac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} dG(\psi)}.$

$$\Longrightarrow \mathbb{E}_w\left(\frac{f}{w}\right) = \frac{\mathbb{E}_1(f)}{\mathbb{E}_1(w)} = \left[\mathbb{E}_f\left(\frac{w}{f}\right)\right]^{-1}.$$

By applying the above formulae to $\pi(\cdot)/r(\cdot) = 1 - \ell(\cdot)/r(\cdot) = 1/\sigma(\cdot) = 1 - 1/\mu(\cdot)$,

Aggregate Labor Cost Share (Average inverse markup rate)	$\dfrac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\dfrac{1}{\mu}\right) = 1 - \left[\mathbb{E}_\pi\left(\dfrac{\mu}{\mu - 1}\right)\right]^{-1} = \dfrac{1}{\mathbb{E}_\ell(\mu)}$
Aggregate Profit Share (Average inverse price elasticity)	$\dfrac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\dfrac{1}{\sigma}\right) = \dfrac{1}{\mathbb{E}_\pi(\sigma)} = 1 - \left[\mathbb{E}_\ell\left(\dfrac{\sigma}{\sigma - 1}\right)\right]^{-1}$
Aggregate TFP	$\ln\left(\dfrac{X}{L}\right) = \ln\left(\dfrac{1}{P}\right) = \ln\left(\dfrac{c}{A}\right) + \mathbb{E}_r[\Phi \circ Z]$

Revisiting Melitz (2003): $\zeta(z) = \sigma > 1 \Leftrightarrow s(z) = \gamma z^{1-\sigma}$

Pricing:	$\mu\left(\frac{\psi}{A}\right) = \frac{\sigma}{\sigma-1} > 1 \Rightarrow \rho\left(\frac{\psi}{A}\right) = 1$

Constant, uniform markup rate; pass-through rate = 1.

Relative firm size:	$\mathcal{E}_r\left(\frac{\psi}{A}\right) = \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1-\sigma < 0.$

Firm size distribution in revenue, profit, employment, never change across equilibriums.

Cutoff Rule:

$$c_0 E\left(\frac{\psi_c}{A}\right)^{1-\sigma} = F,$$

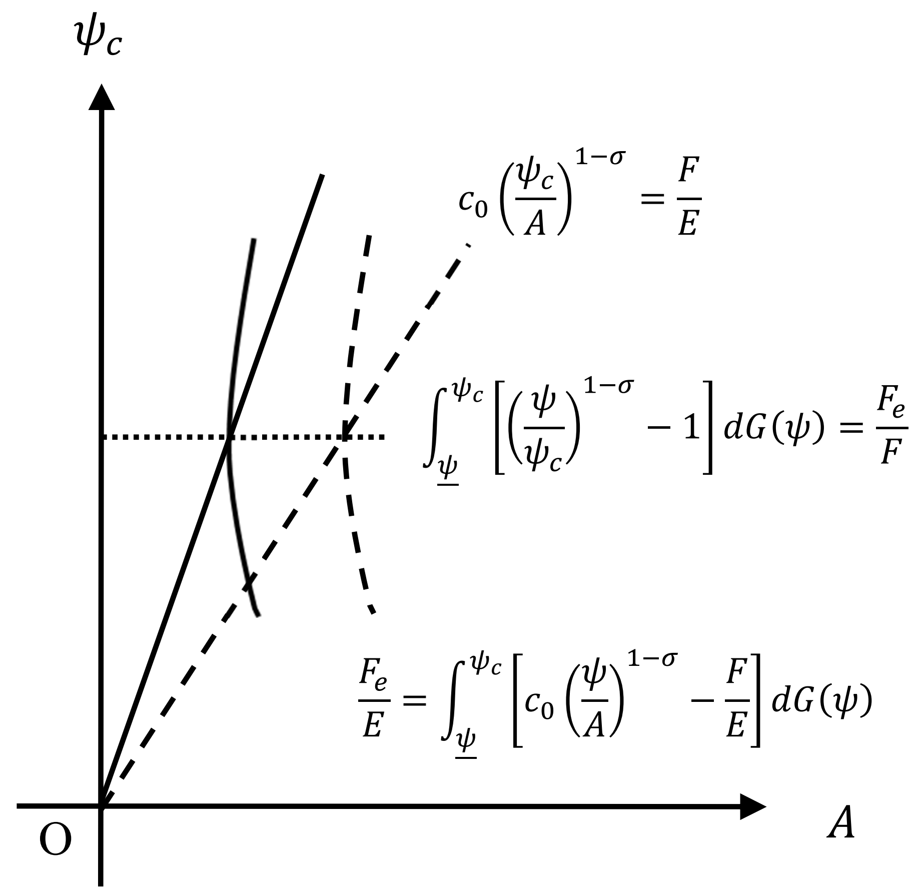
Free Entry Condition:

$$\int_{\underline{\psi}}^{\psi_c}\left[c_0 E\left(\frac{\psi}{A}\right)^{1-\sigma} - F\right] dG(\psi) = F_e,$$

with $c_0 > 0$. As E changes, the intersection moves along

$$\int_{\underline{\psi}}^{\psi_c}\left[\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} - 1\right] dG(\psi) = \frac{F_e}{F}$$

horizontal, i.e., independent of A and of E.



Proposition 1: Under CES,

- $E \uparrow$ keeps ψ_c unaffected; increases both M and $MG(\psi_c)$ *proportionately;* All adjustments at ***the extensive margin.***
- $F_e \downarrow$ reduces ψ_c; increases M; increases (decreases) $MG(\psi_c)$ if $\mathcal{E}_G'(\psi) < (>) 0$; $MG(\psi_c)$ unaffected under Pareto.
- $F \downarrow$ increases ψ_c; increases $MG(\psi_c)$; increases (decreases) M if $\mathcal{E}_G'(\psi) < (>) 0$; M unaffected under Pareto.

Cross-Sectional Implications of the 2^{nd} & 3^{rd} Laws

Marshall's 2nd Law: Cross-Sectional Implications (Proposition 2)

(A2): $\zeta(z_\psi)$ is increasing in $z_\psi \equiv p_\psi/A = Z(\psi/A)$
Note: **A2 ⇒ A1.**

- **Price elasticity** $\zeta(Z(\psi/A)) \equiv \sigma(\psi/A)$, $\sigma'(\psi/A) > 0$; high-ψ firms operate at more elastic parts of demand curve.
  - **Markup Rate**, $\mu(\psi/A)$, decreasing in $\psi/A \Leftrightarrow \mathcal{E}_\mu(\psi/A) < 0$; high-$\psi$ firms charge lower markup rates.
  - **Incomplete Pass-Through:** The pass-through rate, $\rho(\psi/A) = 1 + \mathcal{E}_\mu(\psi/A) < 1$.

- **Procompetitive effect of entry/Strategic complementarity in pricing,** $\frac{\partial \ln p_\psi}{\partial \ln A} = 1 - \rho(\psi/A) = -\mathcal{E}_\mu(\psi/A) > 0$.
  Markups lower under more competitive pressures ($A = A(\mathbf{p})\downarrow$), due to either a larger Ω and/or a lower $\mathbf{p}$

Lemma 5: For a positive-valued function of a single variable, $f(\cdot)$,

$$sgn\left\{\frac{\partial^2 \ln f(\psi/A)}{\partial\psi\partial A}\right\} = -sgn\left\{\mathcal{E}_f'\left(\frac{\psi}{A}\right)\right\} = -sgn\left\{\frac{d^2 \ln f\left(e^{\ln(\psi/A)}\right)}{(d\ln(\psi/A))^2}\right\}$$

$f(\psi/A)$ *log-super(sub)modular* in ψ & $A \Leftrightarrow \mathcal{E}_f'(\cdot) < (>)0 \Leftrightarrow \ln f\left(e^{\ln(\psi/A)}\right)$ concave (convex) in $\ln(\psi/A)$

- **Profit,** $\pi(\psi/A)L$, always decreasing, **strictly log-supermodular** in ψ and A.
  $A\downarrow$ → a proportionately larger decline in profit for high-ψ firms→ Larger dispersion of profit

3rd Law: Cross-Sectional Implications (Propositions 3, 4, and 5)

In addition to **A2**, if we further assume, with some empirical support, e.g. Berman et.al.(2012), Amiti et.al.(2019),

(A3): $\mathcal{E}'_{\zeta/(\zeta-1)}(z) \geq (>)0 \Leftrightarrow \mathcal{E}'_{\mu}(\psi/A) = \rho'(\psi/A) \geq (>)0$. --we call it **Weak (Strong) 3rd Law**.
Under translog, $\rho(\psi/A)$ is strictly decreasing, violating even the weak 3rd Law.

- **Markup rate**, $\mu(\psi/A)$, decreasing under A2, **log-submodular** in ψ & A under A3. For strong A3, it is strict and $A \downarrow$ → a proportionately smaller decline in markup rate for high-ψ firms → smaller dispersion of markup rate

- **Revenue**, $r(\psi/A)E$, always decreasing, **strictly log-supermodular** in ψ & A under *weak A3*
$A \downarrow$ → a proportionately larger decline in revenue for high-ψ firms → Larger dispersion of revenue

- **Employment,** $\ell(\psi/A)E = \frac{r(\psi/A)}{\mu(\psi/A)}E$, *hump-shaped* in ψ/A, **strictly log-supermodular** in ψ & A under *weak A3*
Employment is increasing in ψ across all active firms with a large enough overhead/market size ratio.
$A \downarrow$ → Employment up for the most productive firms.

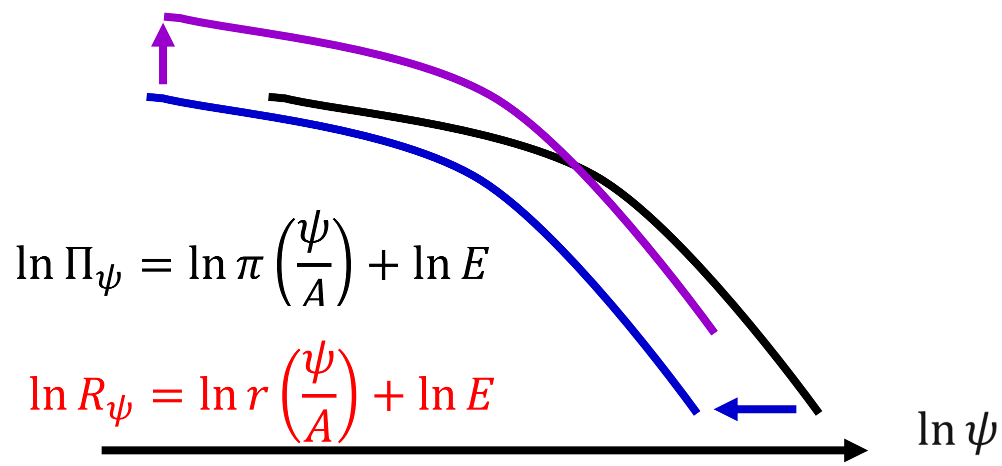
- **Pass-through rate,** $\rho(\psi/A)$, **strictly log-submodular** in ψ & A for a small enough $\bar{z}$ under strong A3
$A \downarrow$ → a proportionately smaller increase in the pass-through rate for low-ψ firms among the active.

Cross-Sectional Implications of More Competitive Pressures, $A\downarrow$: A Graphic Representation

Profit(Revenue) Function: $\Pi_\psi = \pi(\psi/A)E$;, $R_\psi = r(\psi/A)E$

- *always* decreasing in ψ
- strictly log-supermodular *under A2 (Weak A3)*

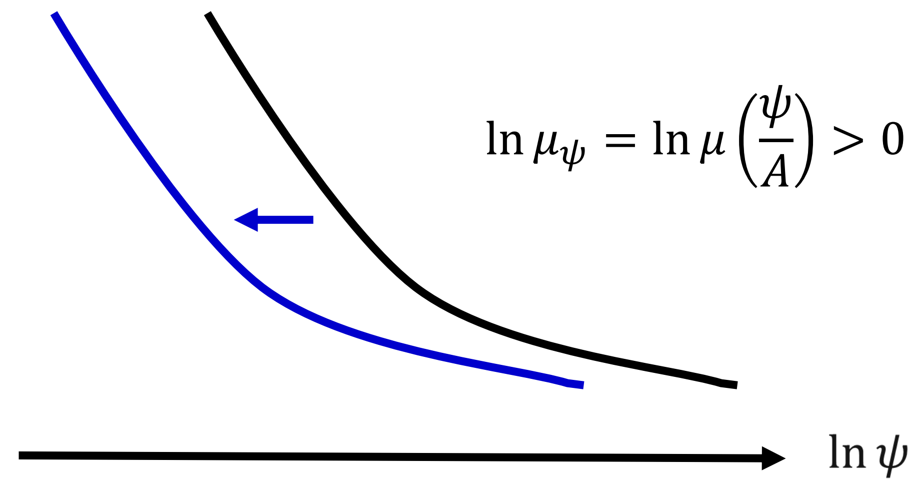
→ $A\downarrow$ with L fixed shifts down with a steeper slope at each ψ;
→ $A\downarrow$ due to $E\uparrow$, a parallel shift up, a single-crossing.



Markup Rate Function: $\mu_\psi = \mu(\psi/A) > 1$

- decreasing in ψ *under A2*
- weakly log-submodular *under Weak A3*
- strictly log-submodular *under Strong A3*

→ $A\downarrow$ shifts down with a flatter slope at each ψ

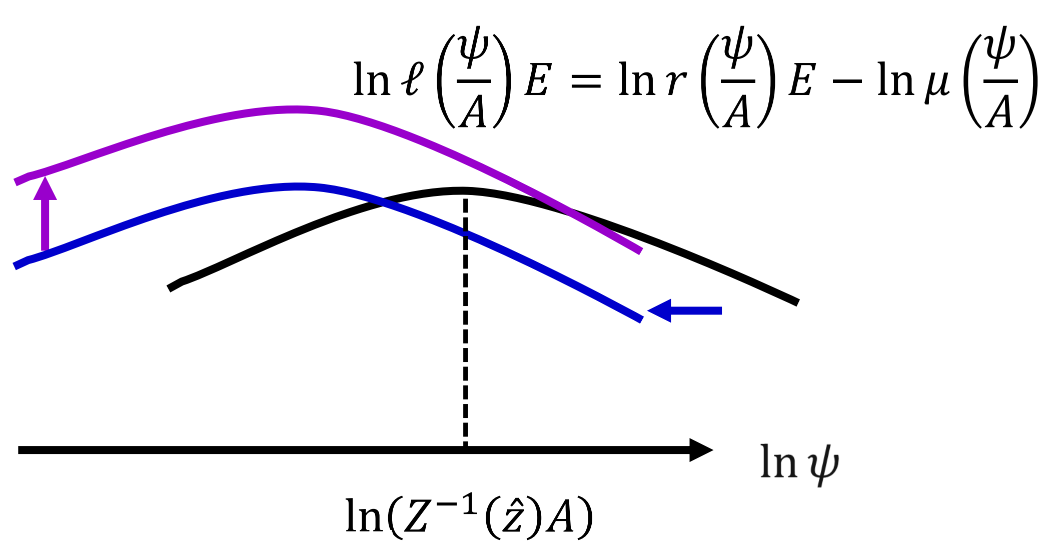


✓ With $\ln\psi$ in the horizontal axis, $A\downarrow$ causes a parallel leftward shift of the graphs in these figures.
✓ $f(\psi/A)$ is strictly log-super(sub)modular in ψ & $A \Leftrightarrow \ln f(\psi/A)$ is (strictly) concave(convex) in $\ln(\psi/A)$.

Employment Function: $\ell(\psi/A)E = r(\psi/A)E/\mu(\psi/A)$

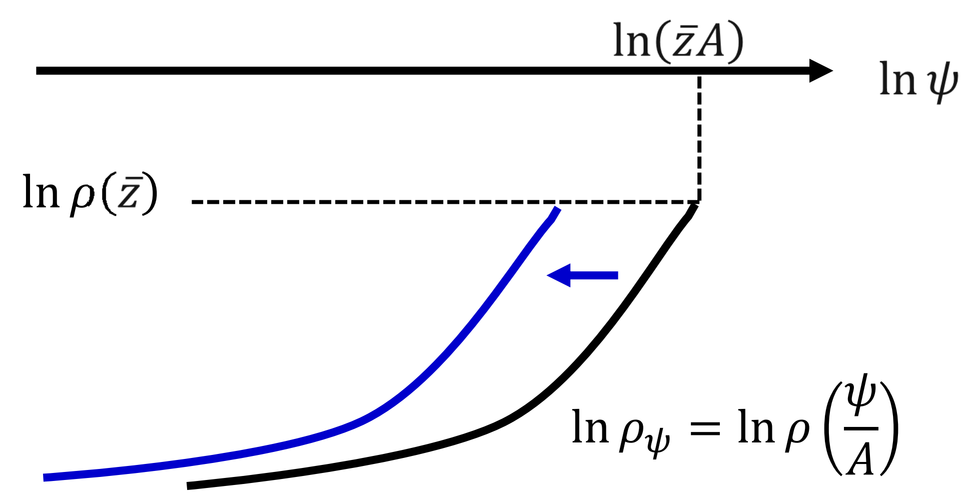
- *Hump-shaped* in ψ *under A2 and weak A3.*
  $\rightarrow A\downarrow$ shifts up (down) for a low (high) ψ with $A\downarrow$
- Strictly log-supermodular *under weak A3*
  for $A\downarrow$ with a fixed L; for $A\downarrow$ caused by $E\uparrow$

Single-crossing even with a fixed E



Pass-Through Rate Function: $\rho_\psi = \rho(\psi/A)$

- $\rho(\psi/A) < 1$ *under A2, hence it cannot be s*trictly log-submodular for a higher range of ψ/A
- Strictly increasing in ψ *under Strong A3*
- Strictly log-submodular for a lower range of ψ/A *under A2 and Strong A3* $\Rightarrow A\downarrow$ shifts up with a steeper slope at each ψ *with a small enough* $\bar{z}$.



In summary, more competitive pressures ($A\downarrow$)

- $\mu(\psi/A)\downarrow$ under A2 & $\rho(\psi/A)\uparrow$ under strong A3
- Profit, Revenue, Employment become more concentrated among the most productive.

Comparative Statics: General Equilibrium Effects

Comparative Statics: General Equilibrium Effects of F_e, E, and F on A and ψ_c

Proposition 6:

$$\begin{bmatrix} d\ln A \\ d\ln\psi_c \end{bmatrix} = \frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} \begin{bmatrix} 1-f_x & f_x \\ 1-f_x & f_x-\delta \end{bmatrix} \begin{bmatrix} d\ln(F_e/E) \\ d\ln(F/E) \end{bmatrix}$$

where

$$\frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} = \frac{1}{\mathbb{E}_\pi(\sigma)-1} = \{\mathbb{E}_r[\mu^{-1}]\}^{-1} - 1 = \mathbb{E}_\ell(\mu) - 1 > 0;$$

The average profit/average labor cost ratio among the active firms

$$f_x \equiv \frac{FG(\psi_c)}{F_e + FG(\psi_c)} = \frac{\pi(\psi_c/A)}{\mathbb{E}_1(\pi)} < 1;$$

The share of the overhead in the total expected fixed cost = to the profit of the cut-off firm relative to the average profit among the active firms

$$\delta \equiv \frac{\mathbb{E}_\pi(\sigma)-1}{\sigma(\psi_c/A)-1} = \frac{\pi(\psi_c/A)}{\ell(\psi_c/A)}\frac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(\pi)} \equiv f_x \frac{\mathbb{E}_1(\ell)}{\ell(\psi_c/A)} > 0.$$

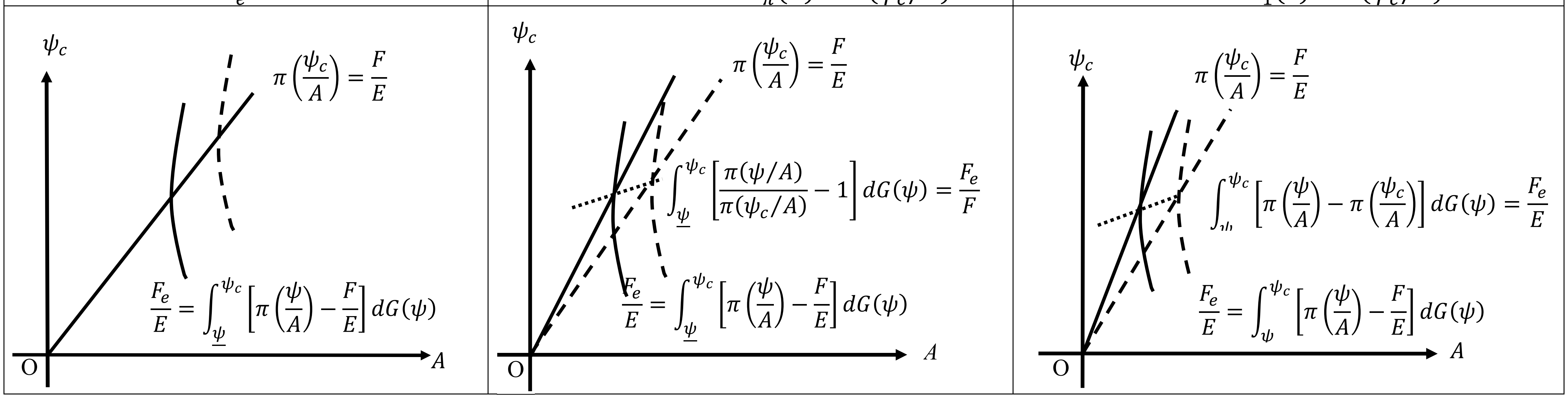
The profit/labor cost ratio of the cut-off firm to the average profit/average labor cost ratio among the active firms.

Corollary of Proposition 6

	A	ψ_c/A	ψ_c
F_e	$\frac{dA}{dF_e} > 0$	$\frac{d(\psi_c/A)}{dF_e} = 0$	$\frac{d\psi_c}{dF_e} > 0$
E	$\frac{dA}{dL} < 0$	$\frac{d(\psi_c/A)}{dE} > 0$	$\frac{d\psi_c}{dL} < 0 \Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma\left(\frac{\psi_c}{A}\right)$, which holds globally if $\sigma'(\cdot) > 0$, i.e., under A2
F	$\frac{dA}{dF} > 0$	$\frac{d(\psi_c/A)}{dF} < 0$	$\frac{d\psi_c}{dF} > 0 \Leftrightarrow \mathbb{E}_1(\ell) < \ell\left(\frac{\psi_c}{A}\right)$, which holds globally if $\ell'(\cdot) > 0$

.

$F_e \downarrow$	$E \uparrow$ under A2 $\Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma(\psi_c/A)$	$F \downarrow$ under $\mathbb{E}_1(\ell) < \ell(\psi_c/A)$



Note: For $F = 0$ & $\frac{\psi_c}{A} = \bar{z} < \infty$, the cutoff rule does not change $E \uparrow$ is isomorphic to $F_e \downarrow$

Market Size Effect on Profit and Revenue Distributions (Proposition 7)

7a: Under **A2,** there exists a unique $\psi_0 \in (\underline{\psi}, \psi_c)$ such that $\sigma\left(\frac{\psi_0}{A}\right) = \mathbb{E}_\pi(\sigma)$ with

$$\frac{d \ln \Pi_\psi}{d \ln E} > 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) < \mathbb{E}_\pi(\sigma) \text{ for } \psi \in \left(\underline{\psi}, \psi_0\right),$$

and

$$\frac{d \ln \Pi_\psi}{d \ln E} < 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) > \mathbb{E}_\pi(\sigma) \text{ for } \psi \in (\psi_0, \psi_c).$$

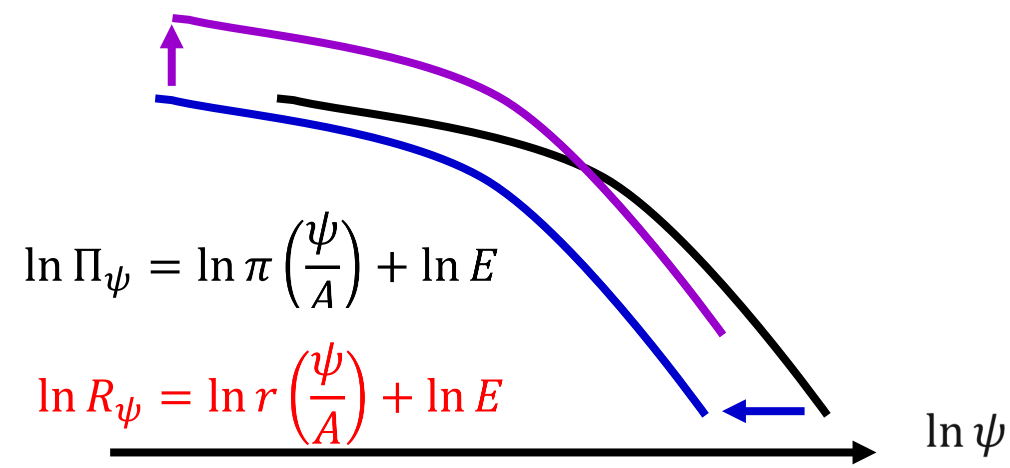
7b: Under **A2 and the weak A3,** there exists $\psi_1 > \psi_0$, such that

$$\frac{d \ln R_\psi}{d \ln E} > 0 \text{ for } \psi \in \left(\underline{\psi}, \psi_1\right).$$

Furthermore, $\psi_1 \in (\psi_0, \psi_c)$ and

$$\frac{d \ln R_\psi}{d \ln E} < 0 \text{ for } \psi \in (\psi_1, \psi_c),$$

for a sufficiently small F.



In short, more productive firms expand in absolute terms, while less productive firms shrink.

The Composition Effect: Average Markup and Pass-Through Rates

- Under A2, $A \downarrow$ causes $\mu(\psi/A) \downarrow$ for each ψ, but distribution shifts toward low-ψ firms with higher $\mu(\psi/A)$.
- Under strong A3, $A \downarrow$ causes $\rho(\psi/A) \uparrow$ for each ψ, but distribution shifts toward low-ψ firms with lower $\rho(\psi/A)$.

Proposition 8: *Assume that $\mathcal{E}_g'(\cdot)$ does not change its sign and $\underline{\psi} = 0$. Consider a shock to F_e, E, and/or F, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of any weighted generalized mean of any monotone function, $f(\psi/A) > 0$, defined by*

$$I \equiv \mathcal{M}^{-1}\left(\mathbb{E}_w\big(\mathcal{M}(f)\big)\right)$$

with a monotone transformation $\mathcal{M}: \mathbb{R}_+ \to \mathbb{R}$ and a weighting function, $w(\psi/A) > 0$, satisfies:

	$f'(\cdot) > 0$	$f'(\cdot) = 0$	$f'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln I}{d\ln A} > 0$	$\frac{d\ln I}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln I}{d\ln A} < 0$
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln A} \gtreqless 0$	$\frac{d\ln I}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln A} \lesseqgtr 0$
$\mathcal{E}_g'(\cdot) < 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln I}{d\ln A} < 0$	$\frac{d\ln I}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln I}{d\ln A} > 0$

Moreover, if $\mathcal{E}_g'(\cdot) = \frac{d\ln(\psi_c/A)}{d\ln A} = 0$, $d\ln I/d\ln A = 0$ for any $f(\psi/A)$, monotonic or not. Furthermore, $\mathcal{E}_g'(\cdot)$ can be replaced with $\mathcal{E}_G'(\cdot)$ in all the above statements for $w(\psi/A) = 1$, i.e., the unweighted averages.

The arithmetic, $I = \big(\mathbb{E}_w(f)\big)$, geometric, $I = \exp[\mathbb{E}_w(\ln f)]$, harmonic, $I = \big(\mathbb{E}_w(f^{-1})\big)^{-1}$, means are special cases.
The weight function, $w(\psi/A)$, can be profit, revenue, and employment.

Corollary 1 of Proposition 8

a) Entry Cost: $f'(\cdot)\mathcal{E}_g'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln F_e} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Longrightarrow \frac{d\ln I}{d\ln E} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln E} \gtreqless 0.$

c) Overhead Cost: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Longrightarrow \frac{d\ln I}{d\ln F} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F} \lesseqgtr 0.$

Furthermore, $\mathcal{E}_g'(\cdot)$ *can be replaced with* $\mathcal{E}_G'(\cdot)$ *for* $w(\psi/A) = 1$, *i.e., the unweighted averages.*

For the entry cost, $\frac{d\ln(\psi_c/A)}{d\ln A} = 0$.

- $\mathcal{E}_g'(\cdot) > 0$; sufficient & necessary for the composition effect to dominate:
    - The average markup & pass-through rates move in the *opposite* direction from the firm-level rates
- $\mathcal{E}_g'(\cdot) = 0$ (Pareto); a knife-edge. $A \downarrow\rightarrow$ no change in average markup and pass-through.
- $\mathcal{E}_g'(\cdot) < 0$; sufficient & necessary for the procompetitive effect to dominate:
  The average markup & pass-through rates move in the *same* direction from the firm-level rates

For market size and the overhead cost, $\frac{d\ln(\psi_c/A)}{d\ln A} < 0$

- $\mathcal{E}_g'(\cdot) > 0$; necessary for the composition effect to dominate:
- $\mathcal{E}_g'(\cdot) \leq 0$; sufficient for the procompetitive effect to dominate:

The Composition Effect: Impact on P/A

$$\ln\left(\frac{A}{cP}\right) = \mathbb{E}_r[\Phi \circ Z]$$

$$\zeta'(\cdot) \gtreqless 0 \implies \Phi'(\cdot) \lesseqgtr 0 \Leftrightarrow \Phi \circ Z'(\cdot) \lesseqgtr 0$$

Corollary 2 of Proposition 8: *Assume $\underline{\psi} = 0$, and neither $\zeta'(\cdot)$ nor $\mathcal{E}_g'(\cdot)$ change the signs. Consider a shock to F_e, E, and/or F, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of P/A satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \implies \frac{d\ln(P/A)}{d\ln A} > 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \implies \frac{d\ln(P/A)}{d\ln A} < 0$
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \gtreqless 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \lesseqgtr 0$
$\mathcal{E}_g'(\cdot) < 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \implies \frac{d\ln(P/A)}{d\ln A} < 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \implies \frac{d\ln(P/A)}{d\ln A} > 0$

Comparative Statics on $MG(\psi_c)$

Proposition 9: *Assume that $\mathcal{E}_G'(\cdot)$ does not change its sign and $\underline{\psi} = 0$. Consider a shock to F_e, F, and/or E, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of the mass of active firms, $MG(\psi_c)$, is as follows*:

$$If\ \mathcal{E}_G'(\cdot) > 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} > 0;$$
$$If\ \mathcal{E}_G'(\cdot) = 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} \gtreqless 0;$$
$$If\ \mathcal{E}_G'(\cdot) < 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} < 0.$$

Corollary 1 of Proposition 9

a) Entry Cost: $\mathcal{E}_G'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F_e} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: $\mathcal{E}_G'(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln E} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln E} > 0.$

c) Overhead Cost: $\mathcal{E}_G'(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F} < 0.$

For a decline in the entry cost,

$\mathcal{E}_g'(\cdot) > 0$ *sufficient & necessary* for $MG(\psi_c) \downarrow$; $\mathcal{E}_g'(\cdot) = 0$, no effect; $\mathcal{E}_g'(\cdot) < 0$; *sufficient & necessary* for $MG(\psi_c) \uparrow$

For market size and the overhead cost

$\mathcal{E}_g'(\cdot) > 0$ *necessary* for $MG(\psi_c) \downarrow$; $\mathcal{E}_g'(\cdot) \leq 0$ *sufficient* for $MG(\psi_c) \uparrow$

Impact of Competitive Pressures on Unit Cost/TFP

By combining Corollary 2 of Proposition 8 and Corollary 1 of Proposition,

Corollary 2 of Proposition 9: *Assume* $\underline{\psi} = 0$, *and neither* $\zeta'(\cdot)$ *nor* $\mathcal{E}_g'(\cdot)$ *change the signs. Consider a shock to* F_e, E, *and/or* F, *which affects competitive pressures, i.e.,* $dA \neq 0$. *Then, the response of* P *satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d \ln P}{d \ln A} > 1 \; for \; F_e$	$\frac{d \ln P}{d \ln A} = 1$	?
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d \ln P}{d \ln A} = 1 \; for \; F_e$ $0 < \frac{d \ln P}{d \ln A} < 1 \; for \; F \; or \; E;$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} = 1 \; for \; F_e$ $\frac{d \ln P}{d \ln A} > 1 \; for \; F \; or \; E$
$\mathcal{E}_g'(\cdot) < 0$	$0 < \frac{d \ln P}{d \ln A} < 1$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} > 1$

Sorting of Heterogenous Firms Across Multiple Markets

Sorting: GE Implications in a Multi-Market Setting

Many markets of different size. Firms, after learning their ψ, choose which market to enter.

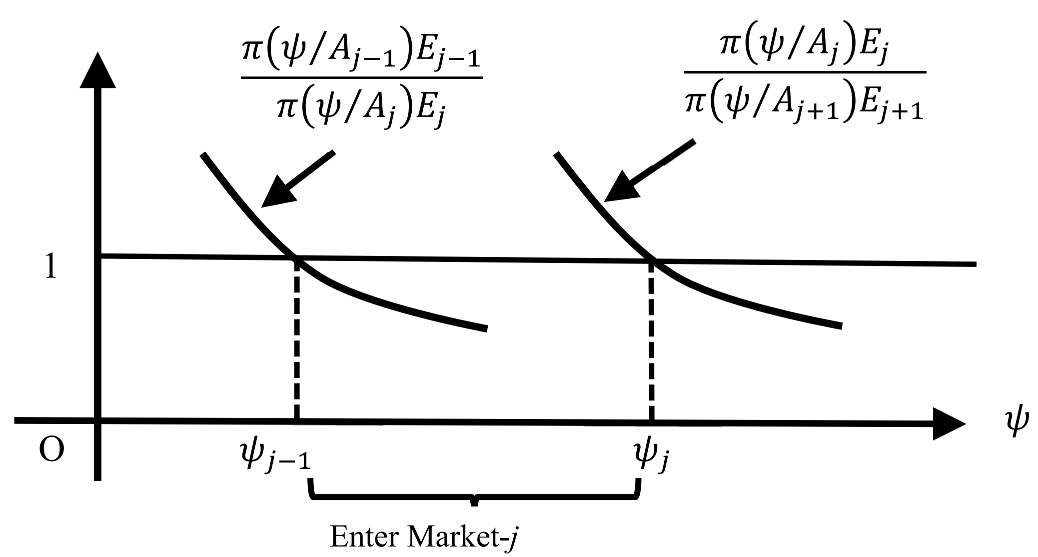
Proposition 10: Assortative Matching

More competitive pressures in larger markets:

$$E_1 > E_2 > \cdots > E_J > 0 \;\Rightarrow\; 0 < A_1 < A_2 < \cdots < A_J < \infty$$

Under A2, more efficient firms sort themselves into larger markets: Firms $\psi \in (\psi_{j-1}, \psi_j)$ entering market-j, where

$$0 \leq \underline{\psi} = \psi_0 < \psi_1 < \psi_2 < \cdots < \psi_J < \overline{\psi} \leq \infty.$$

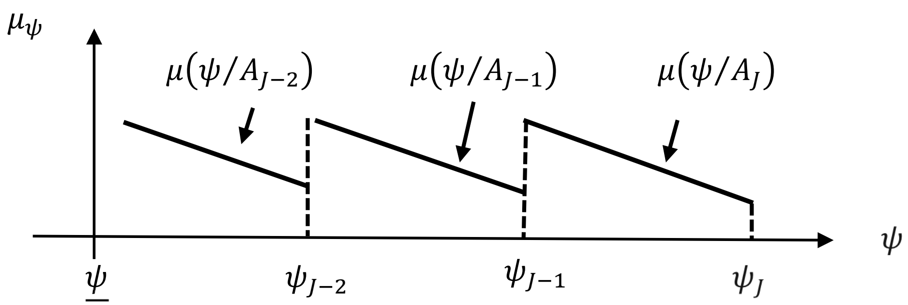


Sorting: GE Implications in a Multi-Market Setting

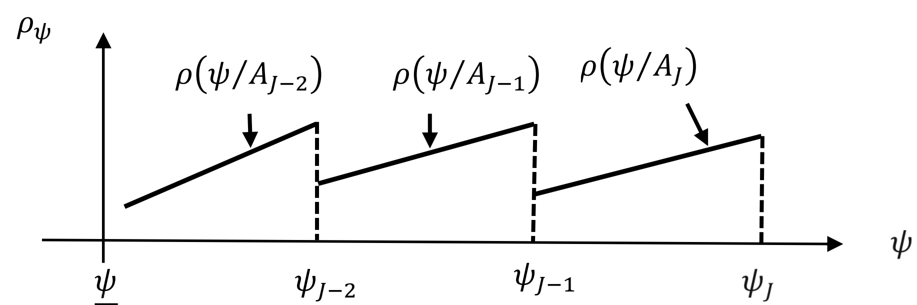
Proposition 11: The Composition Effect: *Examples* with Pareto-productivity such that

- The average markup rates *higher* (the average pass-through rates *lower* under Strong A3) in larger (more competitive) markets
- A decline in F_e causes uniform declines in ψ_j & A_j with the average markup/pass-through rates unchanged.

Markup Rate across markets under A2



Pass-Through Rate across markets under strong A3



A caution against testing A2/A3 by comparing the average markup/pass-through rates in cross-section of cities.

International/Interregional Trade with Differential Market Access

Two Symmetric Markets, characterized by

The same market size E, "Labor" supplied at the same price (equal to one), the numeraire, ensuring the same level of competitive pressures, A.

- After paying F_e, & learning ψ_ω, firm ω can produce its product at home & sell to both markets.
  - The overhead cost, $F > 0$ and the marginal cost of selling to the home market, ψ_ω.
  - The overhead cost, $F > 0$ and the marginal cost of selling to the export market, $\tau\psi_\omega > \psi_\omega$. **Iceberg cost, $\boldsymbol{\tau > 1}$.**

Cutoff Rules: Firm ω sells to both markets iff $\psi_\omega \leq \psi_{xc} < \psi_c$; only to the home market iff $\psi_{xc} < \psi_\omega \leq \psi_c$, where

$$F \equiv \pi\left(\frac{\psi_c}{A}\right)E \equiv \pi\left(\frac{\tau\psi_{xc}}{A}\right)E.$$

Free-Entry Condition:

$$F_e = \int_{\underline{\psi}}^{\psi_c}\left[\pi\left(\frac{\psi}{A}\right)E - F\right]dG(\psi) + \int_{\underline{\psi}}^{\psi_{xc}}\left[\pi\left(\frac{\tau\psi}{A}\right)E - F\right]dG(\psi).$$

These two conditions jointly pin down the equilibrium value of $\psi_c \equiv \tau\psi_{xc} \equiv \pi^{-1}(F/E)A$ by:

$$\frac{F_e}{E} = \int_{\underline{\psi}}^{\psi_c}\left[\pi\left(\frac{\psi}{\psi_c}\pi^{-1}\left(\frac{F}{E}\right)\right) - \frac{F}{E}\right]dG(\psi) + \int_{\underline{\psi}}^{\psi_c/\tau}\left[\pi\left(\frac{\tau\psi}{\psi_c}\pi^{-1}\left(\frac{F}{E}\right)\right) - \frac{F}{E}\right]dG(\psi).$$

Globalization Effect

After solving for $\psi_c \equiv \tau\psi_{xc} \equiv \pi^{-1}(F/E)A$, the mass of entering firms, M, and hence those of active firms $MG(\psi_c)$, and of exporting firms, $MG(\psi_{xc})$, are pinned down by:

Adding-Up (Resource) Constraint:

$$M\left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right) dG(\psi) + \int_{\underline{\psi}}^{\psi_{xc}} r\left(\frac{\tau\psi}{A}\right) dG(\psi)\right] = 1.$$

Proposition 12: The Effect of Globalization: A Reduction in $\tau > 1$.

- A decline in ψ_c and an increase in $\psi_{xc} = \psi_c/\tau$. → $G(\psi_c)$ falls, $G(\psi_{xc})$ rises, and $G(\psi_{xc})/G(\psi_c)$ rises.
- A decline in A and an increase in A/τ. →
  - $r(\psi_\omega/A)$ & $\pi(\psi_\omega/A)$ decline, $r(\tau\psi_\omega/A)$ & $\pi(\tau\psi_\omega/A)$ rise.
  - $\mu(\psi_\omega/A)$ declines and $\mu(\tau\psi_\omega/A)$ rises **under the 2nd law.**
  - $\rho(\psi_\omega/A)$ rises and $\rho(\tau\psi_\omega/A)$ declines **under the Strong 3rd law.**

Three Parametric Families of H.S.A. (Appendix D)

Generalized Translog For $\eta > 0, \sigma > 1$	$s(z) = \gamma\left(-\frac{\sigma-1}{\eta}\ln\left(\frac{z}{\bar{z}}\right)\right)^{\eta}$; $z < \bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}}$	$1 - \frac{1}{\zeta(z)} = \frac{\eta}{\eta - \ln\left(\frac{z}{\bar{z}}\right)} \Rightarrow \begin{matrix}\mathcal{E}_{\mu}(\cdot) < 0 \\ \mathcal{E}_{\mu}'(\cdot) = \rho'(\cdot) < 0\end{matrix}$ satisfying **A2; violating A3.**

Translog is the special case where $\eta = 1$. CES is the limit case, as $\eta \to \infty$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Constant Pass-Through (CoPaTh) For $0 < \rho < 1, \sigma > 1$	$s(z) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}$; $\bar{z} \equiv \beta\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}}$	$1 - \frac{1}{\zeta(z)} = \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}} \Rightarrow \begin{matrix}\mathcal{E}_{\mu}(\cdot) < 0 \\ \mathcal{E}_{\mu}'(\cdot) = \rho'(\cdot) = 0\end{matrix}$; satisfying **A2** & weak **A3;** violating strong **A3**

CES is the limit case, as $\rho \to 1$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Power Elasticity of Markup Rate (Fréchet Inverse Markup Rate) For $\kappa \geq 0$ and $\lambda > 0$	$s(z) = \exp\left[\int_{z_0}^{z} \frac{c}{c - \exp\left[-\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa\xi^{-\lambda}}{\lambda}\right]}\frac{d\xi}{\xi}\right]$	$1 - \frac{1}{\zeta(z)} = c\exp\left[\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[-\frac{\kappa z^{-\lambda}}{\lambda}\right]$ $\Rightarrow \mathcal{E}_{\mu}(\cdot) < 0; \mathcal{E}_{\mu}'(\cdot) = \rho'(\cdot) > 0$ satisfying **A2** and strong **A3** for $\kappa > 0$ and $\lambda > 0$.

CES for $\kappa = 0$; $\bar{z} = \infty$; $c = 1 - \frac{1}{\sigma}$; CoPaTh for $\bar{z} < \infty$; $c = 1$; $\kappa = \frac{1-\rho}{\rho} > 0$, and $\lambda \to 0$**.**